Lawrence Woodmere Academy

AP Calculus BC

Dear AP Calculus BC Student,

Welcome to the fun and exciting world of AP Calculus BC. In the upcoming school year, we will be using the concepts that you previously learned in integrated algebra, geometry, algebra II, trigonometry and pre-calculus to expand your knowledge into the world Calculus. To be able to move forward in BC Calculus, you must have a strong foundation in trigonometry, concepts involving functions, and be able to create models from word problems. This summer assignment is designed to allow you to continue to practice these skills and concepts throughout the time that school is not in session.

The AP Calculus BC Summer Assignment packet will not require a lot of time, but it is lengthy enough that you will want to manage your time appropriately. The whole assignment should not be completed at the end of this school year, but should be worked on all summer to keep the material fresh in your mind. As AP Calculus BC students, you will need to be able to manage your time appropriately.

This summer assignment is composed of two sections that review the old material from your years in high school. The first section is a review of trigonometry. Make sure that you make yourself familiar with all the exact values for trigonometric functions on the interval $[0,2\pi]$. There is a NO calculator section on the AP and you must have these values memorized. The second section focuses on a review of functions and modeling. You are expected to answer all questions on a separate sheet of paper and hand in the assignment on the first day of school. All work must be shown for each of the questions and you must provide explanations for all multiple choice questions. It's not sufficient enough to get the right answer, but you must be able to explain your answer as well. The assignment will be graded for completion and effort.

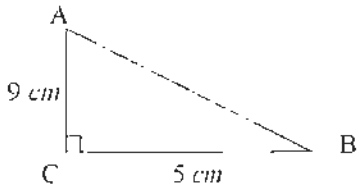
You should also get the required supplies for the course which includes graph paper, notebook, pencil, and a TI-83 or TI-84. You may also want to go to a bookstore this summer and pick up an AP preparation guide for the AP exam (I recommend Baron's).

Again, welcome to AP Calculus BC!

Good luck and I look forward to seeing you in September.

TRIGONOMETRY:

1. Use the diagram on the right to find the exact values of the following:



   a. tan *A*

   b. cos *B*

2. Which of the following is equal to $\csc\theta$?

   a. $\frac{1}{\sin\theta}$

   b. $\frac{1}{\cos\theta}$

   c. $\frac{1}{\tan\theta}$

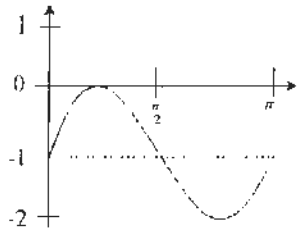
   d. $\frac{1}{\sec\theta}$

3. Find the exact value of $\sec 300°$.

4. Find the reference angle for an angle measuring $145°$.

5. Which of the following graphs represents the equation $y = -1 + \sin 2x$ over a one-period interval?

a.



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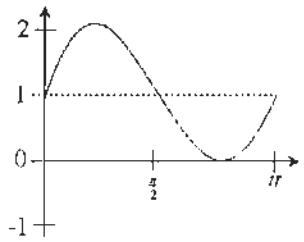

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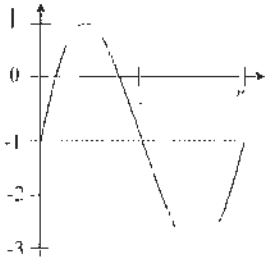
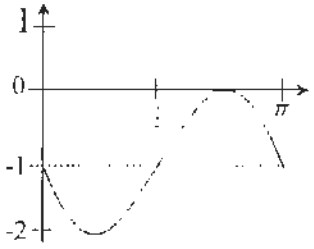


d.



6. Change $125°$ to radian measure.

7. Change $\frac{25\pi}{18}$ radians to degree measure.

8. Find the exact value of each of the following:

a. $\sin\frac{2\pi}{3}$ b. $\tan\frac{\pi}{2}$ c. $\cos\frac{\pi}{4}$

d. $\sec\pi$ e. $\csc\frac{3\pi}{2}$ f. $\cot\frac{-\pi}{6}$

g. $\sin^{-1}\frac{-\sqrt{3}}{2}$ h. $\cos^{-1}\frac{\sqrt{2}}{2}$ i. $\tan^{-1}\sqrt{3}$

**You must know the exact values of trigonometric functions by heart. There is a portion of the AP Calculus exam that does NOT allow calculators and these values are necessary in that section. You should have them memorized by the beginning of the school year.

FUNCTIONS:

1. Given that $f(x) = 2x^2 + x - 5$, find $f(-3)$.

2. For $f(x) = x + 5$ and $g(x) = 3x + 1$, find the domain of $\frac{f}{g}$.

a. $(-\infty, \infty)$

b. $\left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \infty\right)$

c. $(-\infty, 5) \cup (5, \infty)$

d. $\left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, 5\right) \cup (5, \infty)$

3. Find the domain for $f(x) = \sqrt{2x + 5}$

a. $\left(-\infty, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, \infty\right)$

b. $\left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$

c. $\left(-\infty, -\frac{5}{2}\right]$

d. $\left[-\frac{5}{2}, \infty\right)$

4. For $f(x) = 2x - 5$ and $g(x) = x^2 - 6$ find $f(g(x))$

a. $4x^2 - 20x + 19$

b. $2x^2 - 11$

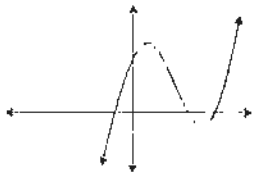
c. $2x^2 - 17$

d. $2x - 11$

5. Write the equation of the vertical asymptote of $f(x) = \frac{4-x}{2+x}$.

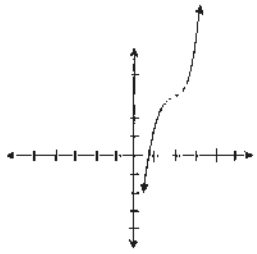
6. The graph of the equation $\frac{x^2+3x-10}{x-2}$ is a line with a hole in it. At what point does the hole occur?

7. Identify the possible formula for the graph shown on the right.



a. $y = (t + 1)(t - 4)(t - 3)$
b. $y = (t - 1)(t - 4)(t + 3)$
c. $y = (t + 1)(t + 4)(t + 3)$
d. $y = -(t + 1)(t + 4)(t - 3)$

8. Which formula best matches the graph shown on the right.



a. $y = (x^2 - 3) + 2$
b. $y = (x - 2)^3 + 3$
c. $y = (x^3 + 3) - 2$
d. $y = (x - 3)^3 + 2$

9. For the function $g(x) = -8 + 4x + 3x^2 - x^3$, what is the leading coefficient?

10. Find the roots of the following equation. Give the values in exact form.

$$x^3 + 24x = 25x^2$$

11. Find the vertex for the following equation.

$$x^2 - 8x - y + 18 = 0$$

12. Find a formula for the inverse of
    a. $f(x) = 4x + 3.$
    b. $f(x) = \frac{x}{3x+1}$

13. Suppose $8000 is invested at a 4% interest rate, compounded monthly. How much will the investment be worth after 9 years?

14. Evaluate the logarithm: $\log_2 \frac{1}{8}$

15. Simplify using the rules of logarithms: $\log_2 25 + \log_2 3$
    a. $\log_4 28$
    b. $\log_2 \frac{25}{3}$
    c. $\log_2 75$
    d. $\log_2 28$

16. Expand the following as sums and/or differences of simpler logarithmic expressions.

$$ln \frac{3x\sqrt{x}}{(2x + 1)^2}$$

a. $\ln 3x + \frac{1}{2}\ln x - 2\ln(2x + 1)$
b. $2\ln(2x + 1) - \ln 3x + \frac{1}{2}\ln x$
c. $3\ln x + \ln\sqrt{x} - 2\ln(2x + 1)$
d. $\ln 3x + 2\ln x - \frac{1}{2}\ln(2x + 1)$

17. Convert to an exponential equation: log x = 15

a. $e^{15} = x$

b. $10^{15} = x$

c. $15^{10} = x$

d. $x^{15} = 10$

18. The population of bacterial culture doubled in 8 hours. What was the exponential growth rate?

a. 3.8%

b. 4.2%

c. 5.5%

d. 8.7%

19. Find the indicated term of the geometric sequence 100, 80, 64, ... , a_6

a. $\frac{16,384}{625}$

b. $\frac{1,024}{25}$

c. $\frac{4,096}{125}$

d. $\frac{8,192}{25}$

20. Find the sum of the first 36 terms in the arithmetic series: −0.2 ??0.3 ??0.8 ??...

a. 318.6

b. 332.2

c. 307.8

d. 314

21. Find the x and y intercepts for each graph.

a. $y = 2x - 5$

b. $y = x^2 + x - 2$

c. $y = x\sqrt{16 - x^2}$

d. $y^2 = x^3 - 4x$

22. Find the intersection points of the graphs for the given equations

a. $x + y = 8$
$4x - y = 7$

b. $x^2 + y = 6$
$x + y = 4$

c. $x = 3 - y^2$
$y = x - 1$

d. $y^2 = 1 - x^2$
$y^2 = x^2 - 3x + 2$

23. If a and h are real numbers, find and simplify $f(a), f(a + h), \frac{f(a+h)-f(a)}{h}$ when:

a. $f(x) = x^2 - x + 3$

b. $f(x) = \frac{1}{x}$

24. Divide by using **long division.**

a. $(20x^2-13x+2)\div(4x-1)$ b. $(x^2-2x+3)\div(x+5)$

c. $(x^3+2x^2-x-2)\div(x+2)$ d. $(6x^2-7x-5)\div(3x-5)$

25. Divide by using **synthetic division.**

a. $(7x^2-23x+6)\div(x-3)$ b. $(x^4-5x+10)\div(x+3)$

c. $(2x^2+13x-8)\div(x-\frac{1}{2})$ d. $(x^4+6x^3+6x^2)\div(x+5)$

26. a. If $f(x)=3x^{20}-5x^{15}+x^2-2$ is divided by $x+1$, find the remainder.

b. If $f(x)=3x^{20}-5x^{15}+x^2-2$ is divided by $x-1$, find the remainder.

27. Solve the following equations for y'

a. $2x+2xy'=2y+3y^2y'$

b. $\frac{x}{y'+y}=\frac{5}{y'}$

28. Find the zeroes of the function (algebraically): $f(x)=-x^2e^{-x}+2xe^{-x}$

29. Triangle *ABC* has vertices A(0,0), B(4,8), C(10,0)

a. Find the coordinates of M, the midpoint of segment $\overline{AB}$
b. Find the equation of the line that contains M and is parallel to segment $\overline{BC}$
c. Find an equation of the line through points C and M. Is this line perpendicular to bisector $\overline{BC}$

30. Use points (-2, 4) and (6,2)

a. Find the slope of the line containing these points.
b. Find the length of the segment that connects these two points
c. Find the midpoint of the line segment that connects the points

31. Find the line that passes through (-2,4) and the point of intersection of the lines x + 3y = 1 and 2x – y = 5.

32. Describe how each of the following graphs compare to its base graph

a. $y = 2(x + 2)^2 - 4$
b. $y = -2|x - 3| + 6$
c. $y = \frac{3}{x-1} + 4$
d. $y = \frac{1}{2}\sqrt{3 - x} + 1$

33. Find the vertex and the axis of symmetry for the parabola $y = 2x^2 + 8x + 5$

a. Rewrite the graph in vertex form $y = z(x - h)^2 + k$

b. Suppose this graph is shifted 3 units left and 2 units up rewrite the new graph in both vertex form and standard form.

34. You need a Lear Jet for one day. Knowing that Swissair rents a Lear jet with a pilot for $3000 a day and $1.25 per mile, while Air France rents a Lear jet with a pilot for $2500 a day and $2.10 per mile, find the following

a. For each company, write a formula giving the cost as a function of the distance traveled
b. At what Mileage is the price going to be the same for both Swissair and Air France
c. What can you conclude from this?

35. In 1984, the Fizzy Cola Co. sold 23 million gallons of soda. By 2003, the company was selling 127 million gallons of soda. What is the average rate of change in number of gallons of soda per year?

36. The sides of a rectangle a x and 3-2x. Express the rectangle's area as a function of x. Express the rectangle's perimeter as a function of x. Why can x not equal 2?

37. The height and the diameter of a cylinder are equal. Express the volume of the cylinder as a function of its radius.

38. Each leg of an isosceles triangle is twice as long as its base. Express the perimeter of the triangle in terms of the length of the base (b).

AP Calculus Summer Assignment
PLEASE FOLLOW DIRECTIONS GIVEN FOR SUMMER ASSIGNMENT

*Problems with stars next to them should be done **without** a calculator.

Problem 1:

*a. Simplify the expression completely: $3\sqrt{63}$.

*b. Solve the equation for y: $5x-8y=16$.

*c. Multiply the expression: $(2x-5)^2$.

*d. Factor each of these expressions: x^2-5x+6 and x^2-25

*e. Let $y=x^2+2x-3$. Find x when $y=0$.

*f. Solve for x: $(x-2)^2=9$

Problem 2:

*a. If $f(x)=3x^2-x-1$, find $f(0)$, $f(-2)$, $f(2a)$, and $f(a+5)$.

b. Is $x=-1$ a zero of the function $g(x)=x^3-3x^2-x+3$? Why or why not?

c. The sides of a rectangle are x and $3-2x$. Express the rectangle's area as a function of x. Express the rectangle's perimeter as a function of x. Explain why x cannot equal 2.

d. The height and the diameter of a cylinder are equal. Express the volume of the cylinder as a function of its radius.

Problem 3:

*a. Graph $f(x)=x(x-3)(x+1)$. Then tell if the graph of $y=f(x)$ is above or below the x-axis for each of the given set of x-values: $x<-1$; $-1<x<0$; $0<x<3$; $x>3$.

b. Sketch the graphs of $y=x^2-4x+3$ and $x-2y=-6$ on the same set of axes. Find the coordinates of each intersection point.

Problem 3, continued:

c. Solve for x: $4x^4 - 21x^2 + 27 = 0$

Problem 4:

a. For what value of x is the function $g(x) = \dfrac{2x+1}{x+7}$ undefined?

b. Solve $x = y^3 - 4$ for y.

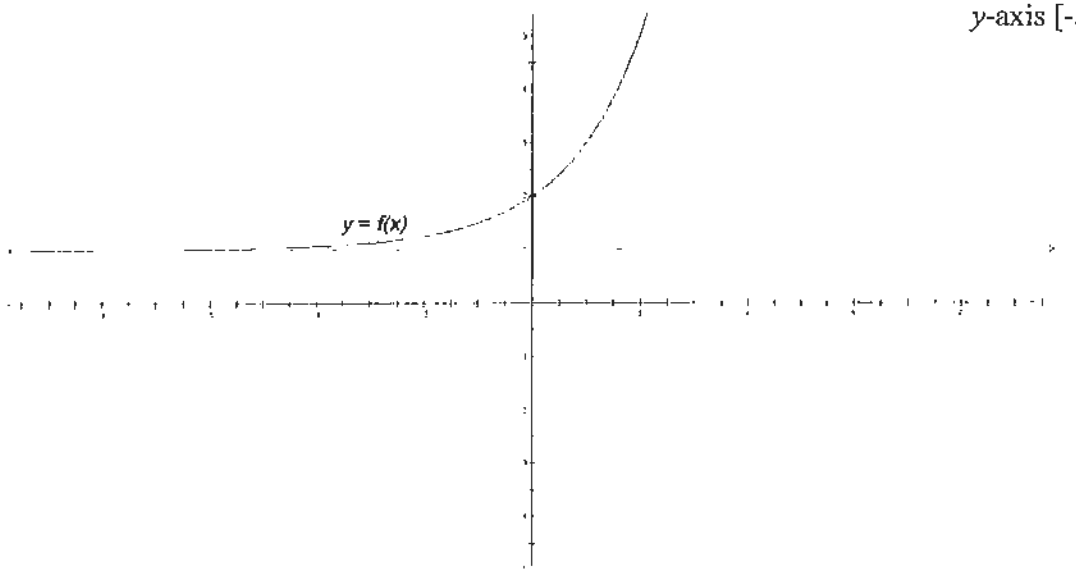
c. Give the dimensions of three different rectangles with the area 6 cm^2.

d. Each leg of an isosceles triangle is twice as long as its base. Express the perimeter of the triangle in terms of the length b of the base.

Problem 5:

*a. Solve for x: $2^x = 64$ $10^x = 1000$ $\left(\dfrac{x}{2}\right)^3 = 125$ $x^5 = 0.00001$

b. Sketch the inverse of the function $y = f(x)$. (Scale: each mark = .5 unit.) x-axis [-10,10]
y-axis [-5,5]

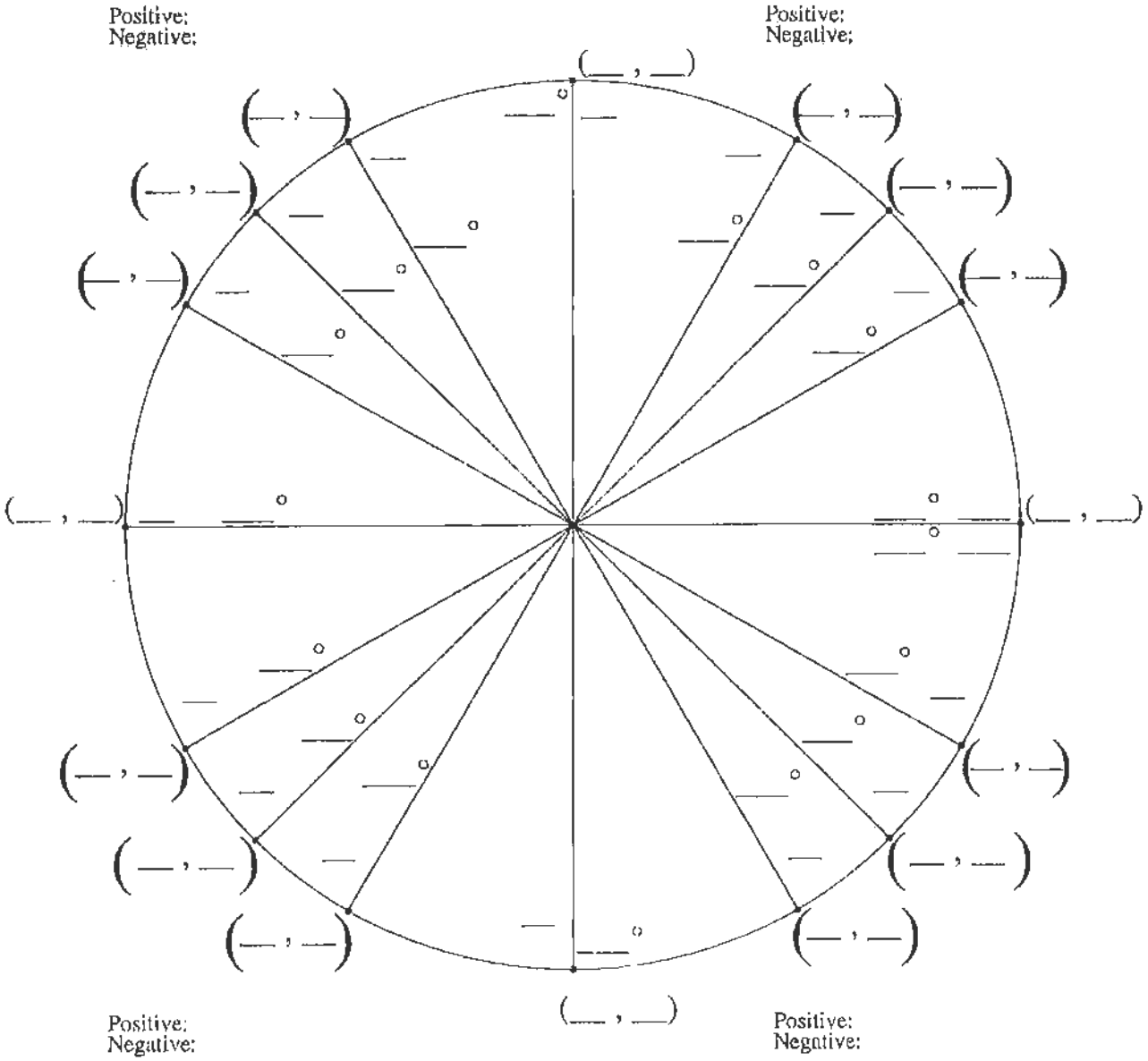


c. Let g be a linear function such that $g(2) = 5$ and $g(6) = -1$. Find and equation for $g(x)$.

d. Find the equation of a line perpendicular to $g(x)$, from part c., and passing through the point (3,-6).

Problem 6:

Fill in The Unit Circle



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Problem 7:

a. Sketch the graph of the circle $x^2+(y-2)^2=25$. Find the circumference and the area of the circle.

b. Find the domain and range: $f(x)=\sqrt{9-x^2}$ $g(x)=-\sqrt{x-3}$

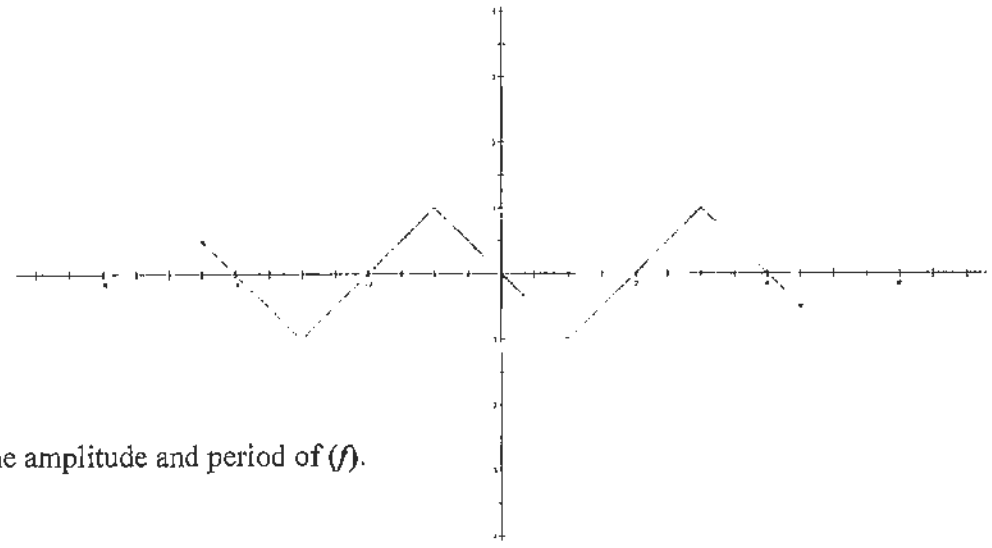
Problem 8:

*a. Graph $y=2\sin(3x)$ for $-2\pi \le x \le 2\pi$.

*b. If $\tan\theta=\frac{5}{2}$ and $\pi<\theta<\frac{3\pi}{2}$, find: $\csc\theta$ $\sec\theta$ $\cot\theta$

*c. Find the exact value of each expression in radians: $Cos^{-1}0$ $Tan^{-1}(1)$

For exercises d and e, refer to the graph of (*f*) below.



*d. Find the amplitude and period of (*f*).

e. Write a description of how the graph above changes based on how the function, $f(x)$, below is changed:

$y=f(2x)$ $y=2f(x)$ $y=f(x)+2$

$y=f(-x)$ $y=f(x-1)$ $y=-f(x)$

In exercise f, g, and h, solve for the variable.

f. $2y=2y(y-1)$

g. Simplify the complex fraction: $\dfrac{\frac{a}{b}-\frac{b}{a}}{a-b}$

h. Solve the system: $y^2=1-x^2$, $y^2=x^2-3x$

Problem 9:

*a. Solve $2+\cos^2\theta = 3\sin^2\theta$ for $0^\circ \le x < 360^\circ$.

*b. Sketch the graph of $y = \tan x$ for $-2\pi \le x \le 2\pi$.

c. Graph $y = -2\cos x$ and $y = \sqrt{x}$ on the same set of axes. How many solutions does the equation $-2\cos x = \sqrt{x}$ have?

Problem 10:

a. Find all real solutions $2x^2 + 2x + 1 = 0$.

b. Plot the point $(x, f(x))$ for $x = 1, 2, 3, 4$, and 5 for $f(x) = 2x + 3$ and $f(x) = \left(-\frac{1}{2}\right)^{x-1}$.

Problem 11:

a. Let $x_1 = 6$, $x_2 = 8$, $x_3 = 9$ and $x_4 = 13$. Evaluate $m = \frac{\sum_{i=1}^{4} x_i}{4}$.

*b. Graph the function $f(x) = e^x$

*c. Graph the inverse, $f^{-1}(x)$, of $f(x) = e^x$.

*d. State the equation of the inverse.

c. Let $f(x) = x^2 - 4$ and $g(x) = \sqrt{x+3}$. Find $(f \circ g)(x)$ and give the domain of the composite function.

f. Graph $y = \frac{x^3}{5}$ and $y = 2^{x-1}$. Between what two integers is a solution of the equation $\frac{x^3}{5} = 2^{x-1}$ located?

Problem 12:

*a. Sketch the graph of each function: $y = 0.5x^2$ $y = 2^{-x} + 2$ $y = \csc x$

b. Solve each equation. Give you answers to the nearest thousandth.

$\log x = 0.72$ $4^x = 3$ $\ln x = 1.09$

c. Express y in terms of x.

$\log y = x + 2$ $\log y = 4\log(x+3)$

Problem 13:

a. Simplify completely: $\dfrac{x^2-9}{x^2+4x+3}$ $\dfrac{x-4}{\sqrt{x-4}}$

b. Solve for x: $(x-2)(x+2)(x-1)^2 = 0$

c. Let $f(x) = \sqrt{x} + 1$. Find $f(1)$ and $f(f(1))$ to the nearest thousandth.

d. Simplify the expression $\dfrac{2(x+h)^2 + 1 - (2x^2+1)}{h}$.

Problem 14:

*a. Simplify. $(16m^4)^{\frac{3}{2}}$ $\dfrac{\left(x^{\frac{1}{2}}y^2\right)^{-\frac{5}{2}}}{x^2y^{\frac{1}{1}}}$

*b. Simplify. $\sqrt{98k^7m^{10}}$ $\sqrt[3]{8a^3b^8}$

Problem 15:

a. Why is $\ln e$ equal to 1? What is the value of e? Where does it come from?

b. Why are there 2π radians in a circle? Draw a picture of a single *radian*.